

Company Secretary Executive Entrance Test (CSEET)

Quantitative Aptitude (Maths)

- CS Vaibhav Chitlangia





## NUMBER SYSTEM

A number system is an arrangement of expressing the numbers in written form. Digits and Symbols in a consistent manner are used in number system. All the numbers are represented in the arithmetic and algebraic structure. The number system inter alia facilitates addition, subtraction, multiplication and division.

### TYPES OF NUMBERS

The various types of numbers including the following:

1. Natural Numbers,
2. Whole Numbers,
3. Integers,
4. Rational Numbers,
5. Irrational Numbers,
6. Real Numbers and etc.

Let us discuss them in detail.

#### 1. Natural Numbers

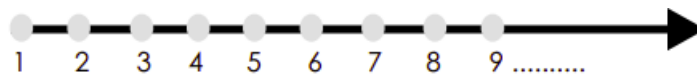
Natural numbers ( $N$ ) are positive numbers i.e. 1, 2, 3 ..... and so one and so forth. Hence counting numbers in natural process like 1, 2, 3, ... constitute the system of natural numbers. These are the numbers which we use in our day-to-day life.

- It has to be noted there is no greatest natural number. For example, if 1 is added to any natural number, we get the next higher natural number, called its successor.
- Four-fundamental operations on natural numbers again generate natural number.

**Four-fundamental operations of Natural Numbers are:**

- Addition (Finding the Sum; '+' ) |
- Subtraction (Finding the difference; '-' ) |
- Multiplication (Finding the product; 'x' ) |
- Division (Finding the quotient; '÷' )

- Addition and multiplication of natural numbers again yield a natural number; but
- subtraction and division of two natural numbers may or may not yield a natural number
- The natural numbers can be represented on a number line as shown below.
- Two natural numbers can be added and multiplied in any order and the result obtained is always same. This does not hold for subtraction and division of natural numbers.



## 2. Whole

## Numbers

When a natural number is subtracted from itself, and one cannot say what is the left out number. To remove this difficulty, the natural numbers were extended by the number zero (0), to get what is called the system of whole numbers.

- Whole numbers do not include any fractions, negative numbers or decimals.
- Again, like before, there is no greatest whole number.
- The number 0 has the following properties:

$$a + 0 = a = 0 + a$$

$a - 0 = a$  but  $(0 - a)$  is not defined in whole numbers  $a$

$$a \times 0 = 0 = 0 \times a$$

- Division by zero (0) is not defined.
- Four fundamental operations can be performed on whole numbers also as in the case of natural numbers (with restrictions for subtraction and division).
- Whole numbers can also be represented on the number line as follows:

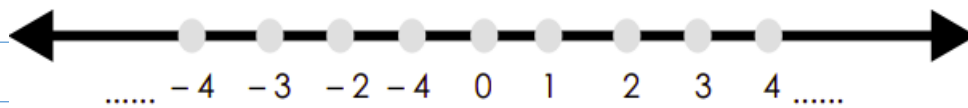


### 3. Integers

It is discovered that subtracting one number from another isn't always possible when working with natural numbers and whole numbers. For instance, in the system of natural numbers and whole numbers, the numbers  $(5 - 7)$ ,  $(6 - 14)$ ,  $(18 - 38)$ , and so on are all impossible. As a result, it required an additional extension of numbers that permit such subtractions. As a result, it is necessary to multiply whole numbers by negative numbers like  $-1$  (also known as negative 1),  $-2$  (also known as negative 2), and so on.  $5 + (-5) = 0$ ,  $10 + (-10) = 0$ ,  $15 + (-15) = 0$ ...,  $99 + (-99) = 0$ , ..

As a result, we have expanded the whole number system to include another set of numbers known as integers. Therefore, the integers are: ...,  $-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, \dots$

- It should be noted that although integers include negative numbers, they are equivalent to whole numbers in every way.
- Examples:  $-3, -2, -1, 0, 1, 2$



Integers can be easily represented on the number line. For example, let us represent  $-5, 7, -2, -3, 4$  on the number line. In the below mentioned figure, the points A, B, C, D and E respectively represent  $-5, 7, -2, -3$  and  $4$ .



**Example 1:** Classify natural numbers, whole numbers and integers among the following:

$-15, 22, -6, 7, -13, 0, 12, -12, 13, -31$

**Example 2:** Simplify the following and indicate whether or not the outcome is an integer.  $12 \times 4$ ,  $7/3$ ,  $18/3$ ,  $36/7$ ,  $14 \times 2$ ,  $18/36$ ,  $13 \times (-3)$

#### 4. Rational Numbers

A number 'r' is called a rational number if it can be written in the form  $p/q$ , where  $p$  and  $q$  are integers and  $q \neq 0$ .

- **When 'a' is a multiple of 'b'**

Suppose  $a = mb$ , where  $m$  is a natural number or integer, then  $a/b = m$ .

- **When a is not a multiple of b**

In this case  $a/b$  is not an integer, and hence is a new type of number. Such a number is called rational number

Thus, a number which can be put in the form  $p/q$ , where  $p$  and  $q$  are integers and  $p/q$ , is called a rational number.

Rational Numbers are of two types:

- **Positive Rational Numbers**

A rational number  $p/q$  is said to be a positive rational number if  $p$  and  $q$  both are either positive integers or negative integers.

Thus  $3/4$ ;  $5/6$ ;  $-3/-2$ ;  $-8/-6$ ,  $-12/57$  are all positive rational numbers.

- **Negative Rational Number**

If the integers  $p$  and  $q$  are of different signs, then  $p/q$  is said to be negative rational number. For example,  $-1/2$ ;  $6/-5$ ;  $-12/4$  and  $16/-3$  are all negative rational numbers.

#### **Concluding Remark:**

(i) Every natural number is a rational number but the vice-versa is not always true.

(ii) Every whole number and integer are a rational number but vice-versa is not always true.

- Irrational Numbers

From the previous discussion, we understand that there may be numbers on the number line that are not rational numbers. Hence, any number that cannot be expressed in the form of  $p/q$ , where  $p$  and  $q$  are integers and  $q \neq 0$ , is an irrational number.

Examples:  $\sqrt{2}$ , 1.010024563...,  $e$ ,  $\pi$

We can also say that decimal numbers in any number line represent irrational numbers. Thus, a decimal expansion which is neither terminating nor is repeating represents an irrational number.

- Real Number

Any number which can be represented on the number line is a Real Number ( $R$ ). It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

TOPIC	DESCRIPTION
Natural Numbers	All counting numbers starting from 1,2,3,4,5.....till infinity. The sum and multiplication product of two natural numbers is always a natural number; however, this doesn't stand are concerning subtraction and division.
Whole numbers	All counting numbers, including 0 (zero). These are also commonly called positive/non-negative integers. Like (0,1,2,3,4,5.....)
Integers	The set of real numbers that consist of all-natural numbers, zero, and their additive inverses. (.....-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.....)

<b>Rational Numbers</b>	All numbers that can be expressed as a ratio between two natural numbers in the form of fractions are called rational numbers. Like $\frac{1}{2}$ , $\frac{1}{3}$ , $\frac{3}{4}$ , etc. All terminable decimals are also rational numbers.
<b>Irrational numbers</b>	Numbers can not be written as fractions, decimals, or ratios. For e.g. Square roots, unending decimals (0.33333333...etc.), pie, etc.
<b>Real numbers</b>	These are numbers that include all of the above types of numbers. Rational, irrational, natural numbers, whole numbers, and so and so forth

### COMPUTATION OF WHOLE NUMBER

Given any natural number, one can add 1 to that number and get the next number i.e. one gets its successor. The successor of 16 is  $16 + 1 = 17$ , that of 19 is  $19 + 1 = 20$  and so on. The number 16 comes before 17, hence it is said that the predecessor of 17 is  $17 - 1 = 16$ , the predecessor of 20 is  $20 - 1 = 19$ , and so on.

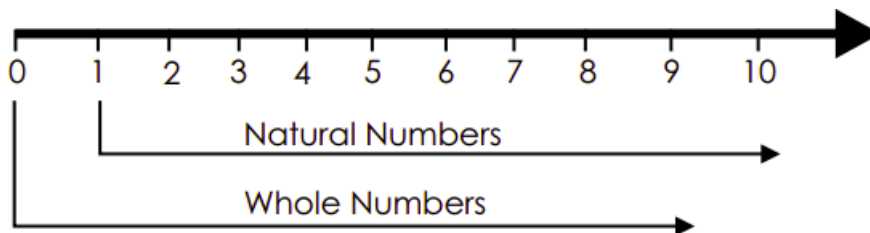
### Facts to be known for Whole Number

- All positive integers, including 0 are whole numbers.
- Real numbers are all whole numbers.
- Whole numbers make up all of nature's numbers. All natural numbers except 0 begin with 1.
- The smallest whole number is zero.
- Natural numbers are regarded whole numbers, although fractions, decimals, and negative numbers are not.
- The number zero is the only one that has no sign.



- g. Fractions are not included in whole numbers because, as the name suggests, a whole number is neither a fraction nor a decimal. The full number is not a fraction as a result, hence it cannot be negative.
- h. Another name for counting numbers is whole numbers.
- i. In mathematics, the numbers 0 through 1, 2, 3, and so forth stand in for the set of Whole Numbers.
- j. The aforementioned facts demonstrate that all whole numbers and natural numbers are components of counting numbers. A whole number can also be obtained from the union of all positive counting integers plus zero.
- k. A smallest whole number is 0 as it starts with zero (0).
- l. The difference between the positive integer number line and negative integer number line is Zero.

#### Whole Number on Number line



#### Properties of Whole Numbers

- **Addition Property:** 0 does not change the final result. For Example-  $2+0 = 2$ .
- **Closure Property:** Two Whole Numbers always produce a Whole Number as their product and their total. For example,  $4 + 10 = 14$  (A Whole Number),  $4 * 10 = 40$  (A Whole Number)
- **Associative Property:** The sum or product of the Whole Numbers remains the same regardless of how the numbers are organised. For example,  $2 * 10 = 20$  and  $10 * 2 = 20$ ,  $2 + 10 = 12$  and  $10 + 2 = 12$ , etc.

- **Multiplication Property:** The outcome of multiplying 1 by a whole number is that number itself. For example  $7 \times 1 = 7$ . If the whole number is multiplied by 0 then the result is 0. For example  $- 7 \times 0 = 0$ .
- **Division Property:** When a whole number is divided by 0, the outcome is ambiguous. . For example-  $7/0 = \text{not defined}$ .
- **Distributive Property:** This property is represented as  $P \times (Q+R) = (P \times Q) + (P \times R)$ . It is applicable for both addition and subtraction. For example - let  $P=11$ ,  $Q=12$ ,  $R=14$ ,  $11 \times (12+14) = (11 \times 12) + (11 \times 14) = 286$ .
- **Commutative Property:**  $P+Q = Q+P$  is a representation of this property. The property also holds for multiplication, but not for division or subtraction. For example -  $P=11$ ,  $Q=12$ ,  $11+12 = 12+11 = 23$ .

### Rounding of the Fractions

- Rounding off finds out the nearest whole number.
- For example: 7 is the closest whole number for 7.3.
- When the decimal number is less than .5, the whole number can be the number below the output.
- When the decimal point is .5 and above than, the whole number would be next whole number after rounding off.

### DECIMAL AND FRACTIONS

#### Key Points on Decimal and Fractions

- A fraction is a number representing a part of a whole.
- This whole may be a single object or a group of objects.
- A fraction whose numerator is less than the denominator is called a proper fraction, otherwise it is called an improper fraction.
- Numbers of the type  $5\frac{1}{3}$ ,  $8\frac{2}{9}$  etc. are called mixed fractions (numbers)

- An improper fraction can be converted into a mixed fraction and vice versa.
- Fractions equivalent to a given fraction can be obtained by multiplying or dividing its numerator and denominator by a nonzero number .
- A fraction in which there is no common factor, except 1, in its numerator and denominator is called a fraction in the simplest or lowest form.
- Fractions with same denominators are called like fractions and if the denominators are different, then they are called unlike fractions.
- Fractions can be compared by converting them into like fractions and then arranging them in ascending or descending order.
- Addition (or subtraction) of like fractions can be done by adding (or subtracting) their numerators.
- Addition (or subtraction) of unlike fractions can be done by converting them into like fractions.
- Fractions with denominators 10,100, etc. can be written in a form, using a decimal point, called decimal numbers or decimals.
- Place value of the place immediately after the decimal point (i.e., tenth place) is  $1/10$ , that of next place (i.e., hundredths place) is  $1/100$  and so on.
- Fractions can be converted into decimals by writing them in the form with denominators 10,100, and so on. Similarly, decimals can be converted into fractions by removing their decimal points and writing 10,100, etc. in the denominators, depending upon the number of decimal places in the decimals. Decimal numbers can be compared using the idea of place value and then can be arranged in ascending or descending order.
- Decimals can be added (or subtracted) by writing them with equal number of decimal places.

### Convert Decimal to Fraction

To convert a Decimal to a Fraction, follow these steps:

- Step 1: Convert 0.50/ 1
- Step 2: Multiply each by 100

$50/100$

- Step 3: Simplify  $50/100$   
 $=1/2$

### Types of Fundamental Arithmetical Operation

Almost all forms of numbers, including integers, fractions, decimals, etc., can be subjected to arithmetic operations. Let's thoroughly comprehend each of the fundamental mathematical processes. The basic arithmetic operations in Mathematics are:

1. Addition (Finding the Sum; '+')
2. Subtraction (Finding the difference; '-')
3. Multiplication (Finding the product; 'x')
4. Division (Finding the quotient; '÷')

### Addition Rules

The following are the addition rules for integers:

- A positive integer is the result of adding two positive numbers.
- A negative integer is created by adding two negative integers.
- When subtracting positive and negative integers, utilize the sign of the biggest integer value.

### Subtraction Rules

The following are the subtraction rules for integers:

- Both the numbers are (+), the answer will also be positive
- Both the numbers are (-), the answer will also be negative
- One number is positive and other number is negative, the answer will be in sign which is largest

### Multiplication Rules

The rules of multiplication are as follows:

- *The product of two positive numbers is positive. In a multiplication if one number is positive and other number is negative, the answer will be negative.*
- *In a multiplication if both the numbers are negative, the answer will be positive.*

#### Division Rules

- *A positive integer is obtained by dividing two positive integers.*
- *A positive integer is obtained by dividing two negative integers.*
- *The negative integer is produced when you divide two integers with different signs.*

SAMPLE QUESTIONS ON WHOLE NUMBERS

1. A number in which one-fifth part is increased by 20 is equal to one-tenth part is increase by 30. Find the number.
  - a) 90
  - b) 100
  - c) 120
  - d) 150
  
2. The Product of two numbers is 150 and the sum of squares of numbers is 325. Find the sum of both numbers.
  - a) 24
  - b) 25
  - c) 29
  - d) 30
  
3. Which is the largest four-digit number divisible by 91?
  - a) 9919
  - b) 9900
  - c) 9909
  - d) None of these
  
4. If the number  $61x4$  is divisible by 6, then what will be the value of  $x$ ?
  - a) 4
  - b) 5
  - c) 6
  - d) 7
  
5. Which of the following numbers is divisible by 13?
  - a) 1235
  - b) 1247

c) 1259

d) 1271

6. Write the smallest whole number.

7. What is the predecessor of whole number 0?

8. Add 30 and 45 and then subtract 20 from the sum.

9. Solve:  $10 + 10 + 10 + 10 + 10$ .

10. Find the value of  $(6 \times 4) \div 12 + 72 \div 8 - 9$ .

11. Simplify:  $24 - 4 \div 2 \times 3$

12. Simplify:  $24 \div 4 \times 3 + 2$

13. Simplify:  $(-20) + (-8) \div (-2) \times 3$

14. Simplify:  $(-5) - (-48) \div (-16) + (-2) \times 6$

15. Simplify:  $52 - (2 \times 6) + 17$

16. Karan purchased 50 computers from a local computer market, only to discover that 10 of them were defective. Can you calculate the Fraction and Decimals of the defective computers in relation to the total computers purchased by Karan?

17. In an 100- office employees, 50 people chose burgers as a snack, while the other employees preferred mango juice. Calculate the Percentage of employees that choose a mango juice and give the result in Decimals.

18. Write  $\frac{1}{2}$ th in Decimals.

19. If  $\frac{3}{2}$  of a number is 9, find the number.

20. Multiply 1.73 and 2.7.

21. Solve the following:

(a)  $4 - \frac{1}{3}$

(b)  $6 + \frac{3}{7}$

22. The product of two numbers is 1.178. If one of them is 0.49, find the other number.

23.  $\frac{1}{4}$  of a number equals  $\frac{4}{5} \div \frac{1}{10}$ . What is the number? (NCERT Exemplar)

24. Simplify the following:

$$(i) \frac{2\frac{1}{2} + \frac{1}{5}}{2\frac{1}{2} \div \frac{1}{5}} \quad (ii) \frac{\frac{1}{4} + \frac{1}{5}}{1 - \frac{3}{8} \times \frac{3}{5}}$$

25. Convert  $5\frac{1}{2}$  into a decimal fraction.

26. Convert 9.9 into a decimal fraction.

27. If half of one-third of a number is 20, then half-tenth of that number will be:

1. 25

2. 50

3. 60

4. 80



28. The difference between a two-digit number and the number obtained by interchanging the positions of its digits is 54. What is the difference between the two digits of that number?

1. 2
2. 5
3. 6
4. None of these

29. The difference between a two-digit number and the number obtained by interchanging the digits is 54. What is the difference between the sum and the difference of the digits of the number if the ratio between the digits of the number is 4:1 ?

1. 4
2. 8
3. 16
4. None of these

30. A two-digit number is such that the product of the digits is 18. When 63 is added to the number, then the digits are reversed. The number is:

1. 19
2. 29
3. 39
4. 49

31. The product of a two digit number is 14. When 18 is added to the number, then the digits interchange their places.

The number is:

- (a) 68
- (b) 59
- (c) 95
- (d) 86

32. If two fifth of one-third of a number is 50, then one-third of that number is:

(a) 376

(b) 3750

(c) 379

(d) 1250



## PERCENTAGE

As per Britannica Dictionary, Percentage, is a relative value indicating hundredth parts of any quantity. One percent (symbolized 1%) is a hundredth part; thus, 100 percent represents the entirety and 200 percent specifies twice the given quantity.

For example, 1 percent of 1,000 Books equals  $1/100$  of 1,000, or 10 Books; 20 percent of the quantity is  $20/100$  of 1,000, or 200. These relationships may be generalized as  $x = PT/100$  where  $T$  is the total reference quantity chosen to indicate 100 percent, and  $x$  is the quantity equivalent to a given percentage  $P$  of  $T$ . Thus, in the example for 1 percent of 1,000 Books,  $T$  is 1,000,  $P$  is 1, and  $x$  is found to be 10.

### Percentage Formula

Percentage formula =  $(\text{Value}/\text{Total value}) \times 100$

Example:  $3/5 \times 100 = 0.6 \times 100 = 60$  per cent

Example: if only 10 of the 200 apples are bad, what percent is that?

As a fraction,  $10/200 = 0.05$

As a percentage it is:

$10/200 \times 100 = 5\%$

1. Calculate 20% of 80.
2. Calculate 25% of 100
3. 30% of 400 apples are bad. How many apples are bad?
4. A Mobile is reduced 25% in price. The old price was Rs. 120. Find the new price.

### Variables of Percentage

Every percentage problem has three possible unknowns or variables :

- Percentage
- Part

- *Base*

*In order to solve any percentage problem, you must be able to identify these variables.*

*Example: 80% of 20 is 16*

- *80 is the percentage.*
- *20 is the base.*
- *16 is the part.*

### *Increase or Decrease Percent*

*We often come across such information in our daily life as.*

*(i) 25% off on MRP*

*(ii) 10% hike in the price of Diesel.*

$$\% \text{ increase} = \frac{[\text{New number} - \text{Original number}]}{\text{Original number}} \times 100;$$

*where, increase in number = New number - original number*

$$\% \text{ decrease} = \frac{[\text{Original number} - \text{New number}]}{\text{Original number}} \times 100$$

*Where decrease in number = Original number - New number*

1. *The price of a laptop was INR 40,000 last year. It has increased by 25% this year.*

*What is the price now?*

### *Finding Discounts in Percentage*

*Discount is a reduction given on the Marked Price (MP) of the article.*

$$\text{So, Discount} = \text{Marked price} - \text{Sale price}$$

1. *A toy marked at Rs. 1,000 is sold for Rs. 920. What is the discount and discount %?*

2. The market price of a Key Board is Rs. 300. A discount of 15% is announced on sales. What is the amount of discount on it and its sale price?

### QUESTIONS ON PERCENTAGE

1. If 20% of 30% of a number is 8, then find the number.
2. Which number is 30% less than 75?
3. The sum of (12% of 28.6) and (5% of 1.75) is equal to what value?
4. A stationery seller had some registers. He sells 40% registers and still has 420 registers. Originally, he had how many registers?
5. Question 10: The strength of a school increases and decreases in every alternate year by 10%. It started with increase in 2000. Then, the strength of the school in 2003 as compared to that in 2000 was?
6. Question 11: A number is increased by 10% and then the increased number is decreased by 10%. The net increase or decrease is?
7. Question 12: During the first year the population of a village is increased by 5% and the second year it is diminished by 5%. At the end of the second year its population was 47880. What was the population at the beginning of the first year?
8. When 75 is added to 75% of a number, the answer is the number. Find 40% of that number.



## RATIO AND PROPORTION

When two parameters are compared, a ratio is created by applying the division operator to the first and second values. The quotient  $x/y$  is typically referred to as the ratio between  $x$  and  $y$  when  $x$  and  $y$  are two parameters of the same type and with similar units, such as  $y$  is not equivalent to 0. The colon (:) symbol is used to denote ratios. It implies that the ratio  $x/y$  can be written as  $x : y$  and has no units. To put it another way, the ratio is the number used to represent one quantity as a fraction of the other item. Only if the two quantities in a ratio have the same unit can they be compared.

A proportion is a mathematical phrase that indicates the two ratios are comparable to one another. The similarity between the two fractional numbers or ratios is, to put it simply, the proportion. The two ratios are intended to be directly proportionate to one another when the two sets of specified quantities are changed in a similar way. The symbol (::) represents proportions and aids in figuring out ambiguous numbers.

### Ratio and Proportion Formula

The Ratio Formula is written as  $x : y \Rightarrow x/y$  where

$x$  = Antecedent or the first term

$y$  = Consequent or the second term

For example, Ratio 8 : 4 is also written as  $8/4$ , where 8 is called the antecedent and 4 is called the consequent.

In order to write a proportion in mathematics for the two ratios,  $a:b$  and  $y:z$  then we express it as  $a:b :: y:z \rightarrow a/b = y/z$

1. The two numbers namely  $b$  and  $y$  are called the **mean terms**.
2. The two numbers namely  $a$  and  $z$  are called the **extreme terms**.



3. In  $a : b = y : z$ , the numbers or parameters of  $a$  and  $b$  should be of the same type with similar units, while  $y$  and  $z$  may be the separate ratios of parameters of the same type with similar units. For example, 10 meter: 20 meter = 50 kg: 100 kg.
4. In the concept of proportion, the product of the mean terms is equivalent to the product of the extreme terms. Hence, we get  $b \times y = a \times z$ .  
For example, In the proportion of two ratios of  $5 : 10 :: 10 : 20$ , we apply the formula of The Product of Mean Terms = The Product of Extreme Terms  
We get,  $10 \times 10 = 5 \times 20 = 100$
5. The proportion formula can be written in the form of  $a/b = c/d$  or  $a : b :: c : d$ .

### Difference between Ratio and Proportion

Sr. No	Ratio	Proportion
(i)	When comparing various quantities with the same units, it is used.	It is used to describe a relationship between two ratios, each of which may have a different set of units. It is used to describe a relationship between two ratios, each of which may have a different set of units.
(ii)	To express a ratio, two symbols are used: a colon (:), and a slash (/).	It is possible to express a proportion using the double colon (::) symbol.
(iii)	It is defined as an expression.	It is termed as an equation.

### Key Notes on Ratio and Proportion

By employing the idea of ratio, any numbers or parameters with comparable units can be compared. Only when two ratios are the same we can say that they are in a proportional relationship. A ratio always produces equivalent outcomes when the individual numbers are multiplied and divided by like numbers.

QUESTIONS ON RATIO & PROPORTION

1. There are 63 students available in the 8th class. The number of students who want to study Sanskrit and the number of students who want to study Mathematics is expressed in the ratio 5:2. Calculate the number of students who want to study Sanskrit and those who want to study Mathematics.
2. R and S started an Electronic shop and decided to divide the profit between them in a ratio of 7:5. The total profit from that shop is Rs. 12,000 by the end of the financial year 2022. What will be the individual profit share for both R and S?
3. If Abhishek travels a distance of 25 km in 5 hours. How much distance can he travel in the time of 8 hours?
4. Calculate the numbers whose sum is 88 and they are written in the ratio of 4:4
5. There are 30 girls and 35 boys in a class. Find the ratio of the no. of boys to the total no. of students.
6. Determine if the following are in proportion.
  - a. 15/45, 40/120
  - b. 32/48, 70/210
7. If the cost of a clothing item is Rs 2170, find the cost of 5m cloth.
8. The weight of 72 books is 9 kg. What is the weight of 40 such books?
9. Determine the proportion of the given ratios. Also, write the means and extremes.  
25cm:1m and rs40:rs160



## SQUARE ROOTS

When an integer is multiplied by itself, the result is known as a square root. The result of a number multiplying itself is referred to as the square number. The symbol for a square root is a  $\sqrt{\quad}$  sign.

### Square Root Definition

Any square root when multiplied by the same number, the result is the original number. For a perfect square number, we obtain perfect square roots.

### Methods to Find Square Root of Numbers

To determine whether a number is a perfect square or an imperfect square, one must first determine the number's square root. A perfect square is defined as a number that can be expressed as the square of the number from the same number system. Imperfect squares are those numbers whose square roots contain fractions or decimals.

Some of the key methods to find out the square root of a number are as follows:

1. Repeated Subtraction Method
2. Prime Factorization
3. Estimation Method
4. Long Division Method

### Repeated Subtraction Method

One of the methods frequently used to determine the square root of a number is repeated subtraction. This approach involves repeatedly subtracting the perfect square number from subsequent odd integers, such as 3, 5, 7, 9, etc., until the result is zero. Starting with 1, the subtraction proceeds through 3, 5, 7, and so forth until 0 is reached. This approach counts how many times the value is deducted from one to get to zero. This count indicates the required square root of the given numbers.

$36 - 1$	$35$
$35 - 3$	$32$

$32-5$	$27$
$27-7$	$20$
$20-9$	$11$
$11-11$	$0$

The sum of the six subtraction operations is 0, as can be seen in the table above. Starting with 1, the subtraction continues until the odd number, 11, is reached. In total, 1, 3, 5, 7, 9 and 11 are deducted. This represents 6 occurrences. 6 is therefore the square root of 36.

#### Prime Factorization Method

The prime factorization method is a simple way to get a number's square root. By dividing the perfect square progressively, this approach divides it into its prime factors. The prime factor pairs are then paired. The square root of the perfect square is obtained by multiplying one element from each pair. Let us find the square root of 196.

The prime factorization of  $196 = 2 \times 2 \times 7 \times 7$ .

When we pair the prime factors and select one from each pair, we have  $7 \times 2 = 14$ . Hence, the square root of 196 is 14.

#### Estimation Method

An approximation method is the square root by estimation method. By making educated guesses about the values, this approach determines the square root of numbers. Taking 4 as an example, the square root is 2, while 9 is the square root, which is 3. Knowing that the square root of 5 will be between 2 and 3 is therefore simple. However, we will still have to check the value of  $\sqrt{6}$  is nearer to 2 or 3.

Let us try finding out the square of 2.4 and 2.9.

The square of 2.4 = 5.76

The square of 2.9 = 8.41

Since the square of 2.4 is 5.76, which is approximately 6, we can say that the square root of 6 is approximately equal to 2.4.

### Properties of Square Root

1. Only a perfect square number can have a perfect square root.
2. An even perfect square has an even square root.
3. The square root of an odd perfect square will be odd.
4. Because a perfect square cannot be negative, it is impossible to define the square root of a negative number.
5. A square root can be found for any number that ends in the digit of the unit, such as 1, 4, 5, 6, or 9.
6. It is impossible to obtain a perfect square root if the unit digit of an integer is 2, 3, 7, or 8.
7. A number cannot have a square root if it has an odd number of zeros at the end. Only an even number of zeros allows for the calculation of a square root.

### Square Root

#### Formula

To determine the square root of a number, use the square root formula. The square root formula is  $y = \sqrt{x}$  to make things easier. It is important to note that  $y \times y = x$ . Here  $x$  is the square of a number  $y$ .

For e.g.,  $2 = \sqrt{4}$ , where  $y = 2$  and  $\sqrt{x} = 4$ , thus  $y \times y = x$ , i.e.  $2 \times 2 = 4$ .

The square root of a perfect square integer like 16 is taken into account. Let's think about the square root of -16 now. The integer -16 has no true square root.  $\sqrt{-16} = \sqrt{16} \times \sqrt{-1} = 4i$  (as,  $\sqrt{-1} = i$ )

Here, "i" is represented as the square root of -1. Hence,  $4i$  is the square root of the number 16.

### QUESTIONS ON SQUARE ROOT

1. Which of the following figures is a square in all its parts?

- a) 111
- b) 225
- c) 142
- d) 156

2. A perfect square number can never have the digit ..... at the units place.

- a) 1
- b) 4
- c) 8
- d) 9

3. Evaluate  $\sqrt{6241}$

- a) 72
- b) 75
- c) 78
- d) 79

4. Find the square root of 6724.

- a) 79
- b) 76
- c) 82
- d) 87





## AVERAGES

In plain English, an average is a single number chosen to represent a group of numbers. This average is typically the arithmetic mean, which is the total of the numbers divided by the number of numbers in the group. The average of the numbers 2, 3, 4, 7, and 9 (which add up to 25) is, for instance, 5. An average could be another statistic like the median or mode depending on the situation. In mathematics, the central value of a set of data is expressed as the average of a list of data. It is defined mathematically as the ratio of the total number of data points to the number of units in the list.

$$\text{Average} = \text{Sum of Values} / \text{Number of values}$$

### Formula to Calculate Average

For a given set of variables, we can quickly calculate the average. Simply add up all the values, then divide the result by the total number of values. Average can be calculated using three simple steps. They are:

1. Step 1: Sum of Numbers:

Finding the sum of all the given numbers is the first step in calculating the average of a set of numbers.

2. Step 2: Number of Observations:

The next step is to determine how many numbers are there in the dataset.

3. Step 3: Average Calculation:

In order to arrive at the average, divide the total by the number of observations. Now, let us consider an example to calculate the average.

### Arithmetic Mean

The most typical kind of average is called the arithmetic mean. The arithmetic mean is the sum of the  $a_i$  as divided by  $n$  where  $n$  is a number. If  $n$  numbers are supplied, each number denoted by  $a_i$  (where  $i = 1, 2, \dots, n$ ), then:

Where,

1.  $n$  is the number of observations

2.  $i$  represent the index of summation  $l$
3. and  $a_i =$  data value for the given index

### Geometric Mean

1. By determining the  $n$ th root of the product of  $n$  numbers, the geometric mean is a technique for determining the central tendency of a set of numbers. In contrast to the arithmetic mean, which is calculated by adding the observations and then dividing the total by the number of observations, it is fundamentally different. However, in the case of the geometric mean, we first calculate the product of all observations before calculating the  $n$ th root of the product, assuming that  $n$  is the number of observations. The formula is given by - Geometric Mean,
2.  $x_1, x_2, x_3, \dots, x_n$  are the individual items up to  $n$  terms

### Harmonic Mean

The reciprocal of the average of the reciprocals of the given data values is referred to as the harmonic mean. The formula to find the harmonic mean is given by:

$$\text{Harmonic Mean, HM} = n / [(1/x_1) + (1/x_2) + (1/x_3) + \dots + (1/x_n)]$$

Where  $x_1, x_2, x_3, \dots, x_n$  are the individual items up to  $n$  terms.

### Average of Negative Numbers

The procedure or formula to calculate the average is the same if the list contains any negative integers. Let's use an example to better grasp this. Example:

Find the average of 3, -7, 8, 12, -2.

**Solution:** The sum of these numbers

$$= 3 + (-7) + 8 + 12 + (-2)$$

$$= 3 - 7 + 8 + 12 - 2$$

$$= 14$$

$$\text{Total Units} = 5$$

$$\text{Hence, average} = 14/5 = 2.8$$

QUESTIONS ON AVERAGE

1. Find the average of 7, 3, 8, 9, 8
2. Find the average of 11, 13, 19, 22, 10
3. If the age of 9 boys in a team is 12, 13, 11, 12, 13, 12, 11, 12, 12. Then find the average age of boys in the team.
4. If the heights of females in a group are 5.1, 5.2, 5.6, 5.4, 5.9, 5.8, 5.10, 5.5, 6, 5.3. Then find the average height.



## INTEREST (SIMPLE AND COMPOUND)

Interest is the extra sum that a borrower pays to a lender in addition to repaying the amount borrowed. For instance, a borrower might take out a loan for Rs. 10,000 and agree to pay an additional Rs. 100 as interest. The sum of interest received or paid over a predetermined period is known as an interest rate. The interest rate, for instance, would be 10% if the prior borrower agreed to pay the debt in full within a year.

Amount of interest you pay or earn depends on the following factors:

1. The rate of interest.
2. The amount of loan.
3. How long does it take to repay loans?

If you use a basic interest formula to determine your interest amount, an interest charge of Rs. 500 will be applied to a loan of Rs. 5000 with an annual interest rate of 10%.

### Formula to Calculate Interest

Interest can be calculated using two methods. These two methods are:

1. Simple Interest
2. Compound Interest

- **Simple Interest**

$$\text{Simple Interest (SI)} = P(\text{Principal}) \times R(\text{Rate of Interest}) \times T(\text{Interest Period}) / 100$$

Here, P stands for the principal sum, R for the rate of interest, and T for the period of interest.

The total amount due in the end is made up of the principal plus the simple interest, or  $P + SI$ . For example,

1. An invested sum fetched a total interest of INR 10000 at the rate of 10% in one year. What was the original principal amount?

- **Compound Interest**

The formula for calculating the amount received when interest is compounded annually:

$$\text{Amount} = \text{Principal} (1 + \text{Rate}/100)$$

The total compounded interest over the term is calculated as

$$\text{Compound Interest} = \text{Amount} - \text{Principal}$$

**Example**

1. In how many years will an amount of Rs. 4000 will be doubled, if the interest rate is 10% per annum?

**Difference between Simple Interest and Compound Interest**

Compound interest is a different kind of interest. Simple interest is based on the principal amount, but compound interest is based on the principal amount along with interest over time. This is the main distinction between simple and compound interest. To further comprehend the idea of simple interest, let's look at a straightforward example.

**QUESTIONS ON INTEREST**

1. Arpit takes an Education loan of Rs 15000 from a bank for a period of 1 year. The rate of interest is 10% per annum. Find the interest and the amount he has to pay at the end of a year.
2. Ankur borrowed Rs 100,000 for 3 years at the rate of 4.5% per annum. Find the interest accumulated at the end of 3 years.

3. Find out the difference between the compound interests on Rs. 5 Lakh for 1 years at 9% per annum compounded quarterly and half-yearly?
4. A sum of money becomes 3 times in 25 years. Calculate the rate of interest.
5. What is the SI on Rs. 7500/- at the rate of 12% per annum for 8 years?
6. A man borrowed Rs 15000/- at the rate of 24% SI and to clear the debt after 6years, how much does he have to return?
7. Maninder took a loan of Rs. 10000 from Prashant. If the rate of interest is 5% per annum compounded annually, find the amount received by Prashant by the end of three years.
8. Richa gave Rs. 8100 to Bharat at a rate of 9% for 2 years compounded annually. Find the amount of money which she gained as a compound interest from Bharat at the end of second year.
9. An amount of Rs 1000 is borrowed at CI at the rate of 2% per annum. What will be the amount to be paid after 3 years if the interest is compounded annually?





## PROFIT AND LOSS

Profit relates to gain; and Loss is the opposite of profit.

1. **Profit (P):** A product is sold at a profit if the price is higher than the cost price. For instance, if a piece of land was bought for Rs. 1,20,000 and sold for Rs. 2,20,000 four years later, there would be a profit of Rs 1 lakh.
2. **Loss (L):** When a product is sold for less than what it costs to produce, the seller suffers a loss. For instance, if a Laptop was purchased for Rs. 50,000 and sold for Rs. 35,000 a year later, the seller would have suffered a Rs 15,000 loss.
3. **Cost Price (CP):** It refers to the price at which a product is made or purchased. It can occasionally additionally cover overhead costs, transportation costs, etc. Shiva, for instance, paid Rs. 20,000 for an Air Conditioner and added Rs. 1500 for shipping and Rs. 2000 for installation. Therefore, the final cost price equals the amount of all completed expenditures, or Rs. 23,500. This cost price is divided into two more categories:
  - a. **Fixed Cost:** Fixed cost is constant as it does not vary with situations.
  - b. **Variable Cost:** It could vary depending on the situation.
4. **Selling Price (SP):** It's the price at which a product is offered for sale. It could be greater than, equal to, or lower than the item's cost price. For instance, if a store owner purchased a table for Rs. 800 and sold it for Rs 1000, the furniture's cost price is Rs 800 and its selling price is Rs 1000.
5. **Marked Price (MP):** Shop owners essentially label this to provide a discount to the customers in such a way that.,
  - a.  $\text{Discount} = \text{Marked Price} - \text{Selling Price}$
  - b.  $\text{Discount Percentage} = (\text{Discount}/\text{Marked price}) \times 100$
6. **Profit Percent (P%):** It is the percentage of profit on the price on which the product was purchased or manufactured.
7. **Loss Percent (L%):** It is the percentage of profit on the price on which the product was purchased or manufactured..

Profit and Loss: Formulae

The profit or gain is equal to the selling price(SP) (-) cost price(CP).

Loss is equal to the cost price (CP)(-) selling price(SP). The formula for the profit and loss percentage is:

$$\text{Profit percentage (P\%)} = (\text{Profit} / \text{Cost Price}) \times 100$$

$$\text{Loss percentage (L\%)} = (\text{Loss} / \text{Cost price}) \times 100$$

Profit	Loss
Cost Price (CP) < Selling Price (SP)	Cost Price (CP) > Selling Price (SP)
Profit = S.P. - C.P.	Loss = C.P. - S.P.
S.P. = C.P. + Profit	C.P. = S.P. + Loss
C.P. = S.P. - Profit	S.P. = C.P. - Loss
Profit % = Profit / C.P. × 100	Loss % = Loss / C.P. × 100

QUESTIONS ON PROFIT AND LOSS

1. Suppose a shopkeeper has bought 1 kg of Mangoes for Rs. 120. And sold it for Rs. 150 per kg. How much is the profit earned by him?
2. Calculate the percentage of the profit gained by the shopkeeper in above situation.
3. A man buys a Cooler for Rs. 2000 and sold it at a loss of 15%. What is the selling price of the Cooler?
4. A Computer is sold at Rs. 12,050 with 15% profit. What would be the gain or loss percentage if it had been sold at Rs. 10,980?
5. Pranav sell an article at a discount of 80% and get a profit of 60% on that article calculate the selling price and the cost price?

6. *By selling a basket for Rs. 19.50, a shopkeeper gains 30%. For how much should he sell it to gain 40%?*
  
7. *Krishnan bought a camera and paid 20% less than its original price. He sold it at 40% profit on the price he had paid. The percentage of profit earned by Krishnan on the original price was?*

